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HEAT EXCHANGE OF A BOREHOLE WITH FROZEN ROCK

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We present a method for calculating the heat exchange of a borehole with the surrounding frozen rock in the presence of solid deposits on the inner walls of the raising column of the borehole.

In many practical problems that occur in construction and maintenance of boreholes in frozen rock it is of interest to know the instantaneous position of the front of phase transition and the law that governs its motion during operation of the borehole [1]. Here, it should be noted first of all that the temperature fields in the system of borehole and surrounding rock are generally nonstationary. However, estimates made [2] show that at the characteristic (for flows in a borehole) rates of change in the temperature fields the influence of their nonstationarity on the heat-transfer coefficient is insignificant. Therefore, the nonstationary process of heat exchange can be calculated by the conventional methods of stationary heat conduction.

To prescribe the intensity of heat exchange between a borehole and the surrounding rock we shall successively consider heat transfer through the walls of the borehole, heat transfer to the surrounding rock, and the intensity of heat exchange between the flow and the borehole wall (or the solid phase on the borehole wall).

Heat Transfer through the Borehole Walls. Oil boreholes usually consist of two coaxial metal pipes, namely: an inner raising column, through which the product of the borehole flows, and an outer casing column. The gap between them is filled with a gas or liquid; in the process of operation this medium can be in a state of thermogravitational convection. The presence and intensity of the thermoconvective flow depend on the Grashof number.

We will assume that the radial temperature fields on the walls and in the gap between the columns are quasistationary and that they satisfy the equation

$$r^{-1}\frac{\partial}{\partial r}\left(\lambda_i r\frac{\partial T_i}{\partial r}\right)=0\,,\quad R_{i-1}<R<R_i\,. \tag{1}$$

Here T_i and λ_i are the temperature and the coefficient of heat conduction in the i-th layer; R_i is the outer radius of the i-th layer. Here $i=1$ is for the raising column, $i=N$ is for the layer that adjoins the rock around the borehole. On the boundaries between the layers the conditions of the equality of temperatures and heat fluxes are to be satisfied:

$$T_i=T_{i+1}\,,\ \ \lambda_i\frac{\partial T_i}{\partial r}=\lambda_{i+1}\frac{\partial T_{i+1}}{\partial r}\,,\ \ r=R_i\,. \tag{2}$$

From Eq. (1) with account for Eqs. (2), for the distribution of the temperature in the i-th layer we obtain

$$T_i=\frac{T_i(R_{i-1})-T_i(R_i)}{\ln(R_{i-1}/R_i)}\ln(r/R_{i-1})+T_i(R_{i-1})\,. \tag{3}$$

For the heat flux through the system of pipes, referred to unit length of the borehole, we write

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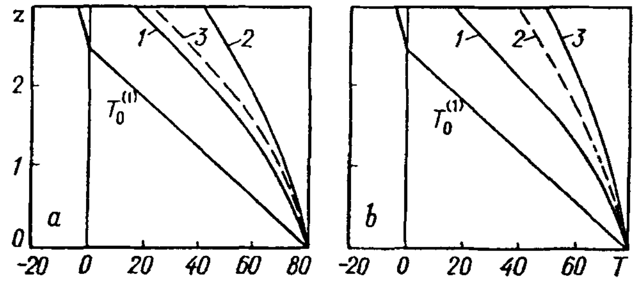

Fig. 1. Distribution of the mean temperature of a gas-liquid flow along a borehole in relation to the kind and state of the material in the intertube space (a) and the thickness dh of heat-insulating material (polyurethane foam, λ = 0.0067 W/(m·K)) on the outer wall of the raising column (b): a: 1, 2, 3) the intertube space is filled with a liquid (oil), a gas, and a gas in a state of thermogravitational convection, respectively; b: 1, 2, 3) dh = 0, 5, 10 mm, respectively, and the remaining volume of the intertube space is filled with oil. z, km; T, °C.

$$Q = -2\pi R_1\lambda_1 \left(\frac{\partial T_1}{\partial r}\right)_{R_1} = -2\pi R_N\lambda_N \left(\frac{\partial T_N}{\partial r}\right)_{R_N}. \tag{4}$$

On the basis of Eqs. (3) and (4) we have

$$Q = 2\pi (T_0 - T_N) \Big/ \sum_{i=1}^{N} \lambda_i^{-1} \ln (R_i/R_{i-1}).$$

From this, for the coefficient of heat transfer through the system of pipes of the borehole we obtain the following expression:

$$\beta = 1 \Big/ R_N \sum_{i=1}^{N} \lambda_i^{-1} \ln (R_i/R_{i-1}) \quad R_N = R_b. \tag{5}$$

Thermogravitational convection of the medium in the space between the pipes is accounted for by means of a factor for the thermal-conductivity coefficient of the medium in the indicated volume [3]:

$$\zeta = 0.049\,(\mathrm{Gr\,Pr})^{1/3}\,\mathrm{Pr}^{0.074}.$$

Figure 1a presents the distribution of the mean temperature of the flow in a borehole as a function of the kind and state of the material in the space between the pipes. It is seen that filling this space in the borehole by a material with a lower thermal-conductivity coefficient exerts a favorable effect on the temperature regime in the shaft of the borehole. But if a gas is in the intertube space in a state of thermogravitational convection, this leads to an increase in heat losses in the borehole. The use of heat-insulated pipes makes it possible to improve substantially the temperature conditions in the raising column (Fig. 1b). Use of boreholes with heat-insulated pipes also increases considerably the delay time of thawing frozen rock [1], and this increases the stability of the borehole and the borehole equipment. The negative aspect of the use of heat-insulated pipes is that their fabrication is rather complex and not always economically justifiable.

Heat Transfer from the Borehole to the Surrounding Rock. When boreholes are used under permafrost conditions, their interaction with the surrounding rock at certain depths is accompanied by thawing of neighboring frozen rock. Therefore, in general to describe the external heat exchange of a borehole it is necessary to solve the problem of heat conduction with account for phase transitions.

If the temperature of the flow in the borehole is higher than the freezing point of water, then in regions around the borehole that are located in permafrost, a zone of thawed rock is formed in a certain interval of time.

Up to the time of appearance of this zone $t^{(1)}$ the borehole will be in contact with frozen rock, and heat transfer between them is described by the heat-conduction equation

$$\frac{\partial T^{(1)}}{\partial t} = \chi^{(1)} r^{-1} \frac{\partial}{\partial r}\left(r \frac{\partial T^{(1)}}{\partial r}\right), \quad 0 < t < t^{(1)}, \quad r > R_b . \tag{6}$$

Up to the beginning of functioning of the borehole we will assume that the temperature around the borehole is uniform and equal to the geothermal one $T_0^{(1)}$. At the boundary of contact of the borehole with the rock we take a condition in the form

$$-\lambda^{(1)} \frac{\partial T^{(1)}}{\partial r} = \beta (T_0 - T^{(1)}), \quad 0 < t < t^{(1)}, \quad r = R_b . \tag{7}$$

At infinity we require the condition of boundedness of the temperature:

$$\partial T^{(1)}/\partial r = 0 , \quad 0 < t < t^{(1)}, \quad r = \infty .$$

Moreover, we write the condition of equality of the fluxes through the outer and inner walls of the borehole:

$$\lambda^{(1)} R_b \left(\frac{\partial T^{(1)}}{\partial r}\right)_{R_b} = \lambda_s R_0 \left(\frac{\partial T_s}{\partial r}\right)_{R_0} . \tag{8}$$

The described external thermal problem for the processes considered is solved rather accurately and effectively on the basis of an integral method [4, 5] according to which the distribution of the temperature around the borehole is taken in the form

$$T^{(1)} = C_1 \ln (r/R_b) + C_2 (r/R_b) + C_3 \tag{9}$$

with the boundary conditions

$$T^{(1)} = T_0^{(1)}, \quad \partial T^{(1)}/\partial r = 0 , \quad r = R_* (t) . \tag{10}$$

The coefficients C_1, C_2, C_3 are determined from the condition that the function (9) must satisfy boundary conditions (7) and (10):

$$C_1 = \tilde{\beta}^{(1)} (T_0^{(1)} - T_0) \tilde{R}_* / (\tilde{R}_* - 1 + \tilde{\beta}^{(1)} [\tilde{R}_* \ln (\tilde{R}_*) + 1 - \tilde{R}_*]) ,$$

$$C_2 = - C_1/\tilde{R}_* , \quad C_3 = T_0^{(1)} + C_1 [1 - \ln (\tilde{R}_*)] , \tag{11}$$

$$\tilde{R}_* = R_*/R_b , \quad \tilde{\beta}^{(i)} = \beta R_b / \lambda^{(i)} , \quad i = 1, 2 .$$

The radius of the thermal effect of the borehole R_* is determined on the basis of Eq. (6). Multiplying this equation by r and integrating from the borehole surface ($r = R_b$) to the boundary of the effect ($r = R_*$), we have

$$\int_{R_b}^{R_*} r \frac{\partial T^{(1)}}{\partial t} dr = \chi^{(1)} \int_{R_d}^{R_*} \frac{\partial}{\partial r} \left(\frac{\partial T^{(1)}}{\partial r}\right) dr ,$$

whence with account for boundary conditions (10)

$$- R_* T_0 \frac{dR_*}{dt} + \frac{d}{dt} \int_{R_b}^{R_*} r T^{(1)} dr = - \chi^{(1)} R_b \left(\frac{\partial T^{(1)}}{\partial r} \right)_{R_b} .$$

Replacing $T^{(1)}$ in this equation by the expression from Eq. (9) and taking conditions (10) into account, we obtain an ordinary differential equation for determining the radius of the thermal effect of the borehole:

$$\frac{d\tilde{R}_*}{dt} = 12 \, (\chi^{(1)}/R_b^2) \, (1 - \tilde{R}_*) \, (\tilde{R}_* - 1 + \tilde{\beta}^{(1)} \, [\tilde{R}_* \ln (\tilde{R}_*) - \tilde{R}_* + 1 \,]) \, /$$

$$/ \, 3 \, [\tilde{\beta}^{(1)} - 7/3\,] + 3\tilde{R}_* \, [2 - \tilde{\beta}^{(1)}\,] + 3\tilde{R}_*^{(2)} \, [1 - \tilde{\beta}^{(1)}\,] +$$

$$+ \, 2 \, [\tilde{\beta}^{(1)} - 3\,] \ln (\tilde{R}_*) - 2\tilde{\beta}^{(1)} \tilde{R}_*^{(3)} \ln (\tilde{R}_*) + \tilde{R}_*^{(3)} \, [3\tilde{\beta}^{(1)} - 2\,]) \, . \tag{12}$$

If we take the following expression for the temperature distribution in a solid layer:

$$T_s = T_0 + \frac{T_\sigma - T_0}{\ln (R/R_0)} \ln (r/R_0) \, , \quad R = R_0 - \delta_s < r < R_0 \, , \tag{13}$$

then on the basis of Eq. (8), considering Eqs. (9) and (10), it is possible to establish the dependence of the temperature of the inner wall of the raising column on the thickness of the solid deposits and on the radius of the effect exerted by the borehole:

$$T_0 = T_\sigma - \frac{B \, (T_0^{(1)} - T_\sigma) \ln (R/R_0)}{\lambda_s/\lambda^{(1)} - B \ln (R/R_0)} \, , \quad B = \frac{\tilde{\beta}^{(1)} \, (\tilde{R}_* - 1)}{\tilde{R}_* - 1 + \tilde{\beta}^{(1)} \, (\tilde{R}_* \ln (\tilde{R}_*) - \tilde{R}_* + 1)} \, .$$

The relations obtained above determine the thermal fields around the borehole until the temperature of the rock on the borehole surface attains the value of the melting temperature of the frozen rock $T^{(1,2)}$. To describe the subsequent process of heat exchange between the borehole and the rock, it is necessary to take account of the presence of the thawed zone between the borehole and the melting surface of the frozen rock of radius $R^{(1,2)}$. In conformity with this, the external thermal problem around the borehole is written in the form

$$\frac{\partial T^{(2)}}{\partial t} = \chi^{(2)} r^{-1} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(2)}}{\partial r} \right) , \quad t > t^{(1)} , \quad R_b < r < R^{(1,2)} \, ; \tag{14}$$

$$\frac{\partial T^{(1)}}{\partial t} = \chi^{(1)} r^{-1} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(1)}}{\partial r} \right) , \quad t > t^{(1)} , \quad R^{(1,2)} < r < \infty \, ; \tag{15}$$

$$- \lambda^{(2)} \frac{\partial T^{(2)}}{\partial r} = \beta \, (T_0 - T^{(2)}) \, , \quad t > t^{(1)} , \quad r = R_b \, ; \tag{16}$$

$$T^{(1)} = T^{(2)} = T^{(1,2)} \, , \quad r = R^{(1,2)} \, ; \tag{17}$$

$$- \lambda^{(2)} \frac{\partial T^{(2)}}{\partial r} + \lambda^{(1)} \frac{\partial T^{(1)}}{\partial r} = \rho^{(1)} l^{(1)} \frac{dR^{(1,2)}}{dt} \, , \quad t > t^{(1)} , \quad r = R^{(1,2)} \, ; \tag{18}$$

$$\partial T^{(1)} / \partial r = 0 \, , \quad r = \infty \, .$$

The distribution of $T^{(1)}$ will be taken in the form of (9). The coefficients C_1, C_2, and C_3 are determined on the basis of boundary conditions (10) and (17):

$$C_1 = (T_0^{(1)} - T^{(1,2)})/(\ln(\tilde{R}_*/\tilde{R}^{(1,2)}) + \tilde{R}^{(1,2)}/\tilde{R}_* - 1),$$
$$C_2 = -C_1/\tilde{R}_*, \quad C_3 = T_0^{(1)} + C_1[1 - \ln(\tilde{R}_*)], \quad \tilde{R}^{(1,2)} = R^{(1,2)}/R_b. \tag{19}$$

Performing calculations similar to those made in deriving Eq. (12), on the basis of Eqs. (15) and (17) with allowance for Eq. (19) we obtain the following ordinary differential equation:

$$-\frac{d\tilde{R}^{(1,2)}}{dt}\left(\frac{\tilde{R}^{(1,2)}}{2\tilde{R}_*}\ln\frac{\tilde{R}_*}{\tilde{R}^{(1,2)}} - \frac{\tilde{R}^{(1,2)}}{4\tilde{R}_*} + \frac{\tilde{R}^{(1,2)^2}}{3\tilde{R}_*^2} - \frac{\tilde{R}_*}{12\tilde{R}^{(1,2)}}\right) +$$
$$+\frac{d\tilde{R}_*}{dt}\left(\frac{\tilde{R}^{(1,2)^2} + \tilde{R}_*^2 + \tilde{R}_*\tilde{R}^{(1,2)}}{6\tilde{R}_*^2}\ln\frac{\tilde{R}_*}{\tilde{R}^{(1,2)}} - \frac{1}{4} + \frac{\tilde{R}^{(1,2)^2}}{4\tilde{R}_*^2}\right) =$$
$$= \frac{\chi^{(1)}}{R_b^2\tilde{R}_*}\left(\ln\frac{\tilde{R}_*}{\tilde{R}^{(1,2)}} + \frac{\tilde{R}^{(1,2)}}{\tilde{R}_*} - 1\right). \tag{20}$$

The temperature profiles in the thawed zone are determined on the basis of the method of successive changes of stationary states [6, 7], according to which we assume that the temperature distribution satisfies the equation

$$r^{-1}\frac{\partial}{\partial r}\left(r\frac{\partial T^{(2)}}{\partial r}\right) = 0, \quad R_b < r < R^{(1,2)},$$

whose solution, with account for boundary conditions (16) and (17), is represented in the form

$$T^{(2)} = T^{(1,2)} + \frac{\tilde{\beta}^{(2)}(T_0 - T^{(1,2)})\ln(\tilde{R}^{(1,2)}R_b r^{-1})}{1 + \tilde{\beta}^{(2)}\ln(\tilde{R}^{(1,2)})}. \tag{21}$$

On the basis of Eqs. (9) and (21) with allowance for Eq. (19) we have

$$\left(\frac{\partial T^{(1)}}{\partial r}\right)_{R^{(1,2)}} = \frac{T_0^{(1)} - T^{(1,2)}}{R_b}\,\frac{\tilde{R}_*/\tilde{R}^{(1,2)} - 1}{\tilde{R}_*\ln(\tilde{R}_*/\tilde{R}^{(1,2)}) + \tilde{R}^{(1,2)} - \tilde{R}_*},$$

$$\left(\frac{\partial T^{(2)}}{\partial r}\right)_{R^{(1,2)}} = \frac{T^{(1,2)} - T_0}{R_b}\,\frac{\tilde{\beta}^{(2)}}{\tilde{R}^{(1,2)}(\tilde{\beta}^{(2)}\ln(\tilde{R}^{(1,2)}) + 1)}.$$

Substituting the above expressions into the Stefan condition (18), we obtain the following ordinary differential equation:

$$\frac{d\tilde{R}^{(1,2)}}{dt} = \frac{\lambda^{(2)}(T_0 - T^{(1,2)})}{R_b^2\rho^{(1)}l^{(1)}}\,\frac{\tilde{\beta}^{(2)}}{\tilde{R}^{(1,2)}(\tilde{\beta}^{(2)}\ln(\tilde{R}^{(1,2)}) + 1)} +$$

$$+\frac{\lambda^{(1)}(T_0^{(1)}-T^{(1,2)})}{R_b^2\rho^{(1)}l^{(1)}}\,\frac{\tilde{R}_*/\tilde{R}^{(1,2)}-1}{\tilde{R}_*\ln(\tilde{R}_*/\tilde{R}^{(1,2)})+\tilde{R}^{(1,2)}-\tilde{R}_*}. \quad (22)$$

Equations (20) and (22) form a system for determining $R_*(t)$ and $R^{(1,2)}(t)$. Thus, the entire external thermal problem for the processes considered in the present work is reduced to the solution of two ordinary differential equations.

Taking into account, just as in the previous case, that

$$\lambda^{(2)}R_b\left(\frac{\partial T^{(2)}}{\partial r}\right)_{R_b}=\lambda_s R_0\left(\frac{\partial T_s}{\partial r}\right)_{R_0},$$

and using Eqs. (13) and (21), for the dependence of the temperature of the inner wall of the raising column on the thickness of the solid deposits and on the thawed-zone radius we obtain the following expression:

$$T_0=T_\sigma+\frac{\bar{\beta}^{(2)}(T^{(1,2)}-T_\sigma)\ln(R/R_0)}{\bar{\beta}^{(2)}\ln(R/R_0)-\lambda_s(1+\bar{\beta}^{(2)}\ln(\tilde{R}^{(1,2)}))/\lambda^{(2)}}.$$

The Intensity of Heat Exchange between the Gas-Liquid Flow and the Wall of the Borehole. To specify the rate of heat exchange between the flow and the wall of the borehole (or the solid phase on the wall of the borehole), it is necessary to single out three segments along the length of the borehole. The first segment is located between the end face and the cross section of the borehole where deposits of solid phase begin to appear on the inner walls of the raising column. For the rate of heat exchange in this region we can write

$$q_w=\beta_w(T-T_0). \quad (23)$$

At the same time, the heat flux determined by expression (23) for this segment of the borehole is equal to the heat flux from the outer wall of the borehole to the surrounding rock:

$$Q=2\pi R_0 q_w=2\pi R_b q,\quad q=-\lambda^{(1)}\left(\frac{\partial T^{(1)}}{\partial r}\right)_{R_b}. \quad (24)$$

Using expression (9) for the distribution of the temperature of the surrounding rock, for the heat flux q we obtain

$$q=\beta_b^{(1)}(T_0-T_0^{(1)}),\quad \beta_b^{(1)}=\frac{\beta(\tilde{R}_*-1)}{\tilde{R}_*-1+\bar{\beta}^{(1)}[\tilde{R}_*\ln(\tilde{R}_*)+1-\tilde{R}_*]}. \quad (25)$$

Substituting Eqs. (23) and (25) into Eq. (24) and solving for T_0, we have

$$T_0=\frac{\beta_w R_0 T+\beta_b^{(1)}R_b T_0^{(1)}}{\beta_w R_0+\beta_b^{(1)}R_b}. \quad (26)$$

On the basis of Eq. (23) with allowance for Eq. (26) we obtain

$$q_w=\frac{T-T_0^{(1)}}{\dfrac{R_0}{R_b\beta_b^{(1)}}+\dfrac{1}{\beta_w}}. \quad (27)$$

The second segment of the borehole is located between the cross section of the start of solid-phase sedimentation and the bottom of frozen rock. Here the rate of heat exchange in the presence of the solid deposits is described by the expression

$$q_w = \beta_w (T - T_\sigma) . \qquad (28)$$

In the third segment, which passes through the zone of frozen rock, heat exchange is prescribed similarly to Eq. (28).

It was assumed in the above argument that the position of the cross section in which solid-phase deposits begin to appear on the inner walls of the raising column is located below the bottom of the frozen rock. But a situation is possible where there are no solid deposits on the inner walls of the borehole up to the lower boundary of the frozen rock. In this case the intensity of heat exchange of the flow with the borehole wall up to the section where solid deposits begin to appear will also be found from Eq. (23), but here formation of a thawed zone around the borehole is to be taken into account in determining the temperature of the inner wall of the raising column. We write an expression similar to Eq. (24), assuming that the outer wall of the borehole is in contact with the thawed zone:

$$Q = 2\pi R_0 q_w = 2\pi R_b q , \quad q = -\lambda^{(2)} \left(\frac{\partial T^{(2)}}{\partial r} \right)_{R_b} .$$

Using expression (21) for $T^{(2)}$, we obtain

$$q = \beta_b^{(2)} (T_0 - T_0^{(1)}) , \quad \beta_b^{(2)} = \beta / (\beta^{(2)} \ln (\tilde{R}^{(1,2)}) + 1) . \qquad (29)$$

Substituting the expressions for q_w and q from Eqs. (23) and (29), we write relations that are similar to Eqs. (26) and (27):

$$T_0 = \frac{\beta_w R_0 T + \beta_b^{(2)} R_b T^{(1,2)}}{\beta_w R_0 + \beta_b^{(2)} R_b} , \quad q_w = \frac{(T - T^{(1,2)})}{\left(\frac{R_0}{R_b \beta_b^{(2)}} + \frac{1}{\beta_w} \right)} .$$

On attaining the cross section where solid deposits begin to appear, we prescribe the heat-exchange rate by expression (28).

During the operation of a borehole a cavitating flow is observed, as a rule, only over a small stretch of it; mainly slug and film modes of flow of the gas-liquid mixture are observed. In the film mode of flow and in the slug mode of flow in a first approximation it is possible to assume that heat transfer from the flow to the borehole wall or to the solid phase is composed of thermal resistances of the gas and the liquid:

$$\beta_w = (1/\beta_{w(liq)} + 1/\beta_{w(g)})^{-1} .$$

In the case where the main portion of the liquid flows in the form of a laminar film along the wall of the channel, we write the following expression for the coefficient of heat transfer from the liquid $\beta_{w(liq)}$:

$$\beta_{w(liq)} = \lambda_{liq} / \delta_{liq} , \quad \delta_{liq} = \frac{R}{2} (1 - \alpha) .$$

The coefficient of heat transfer between the gas core and the liquid film $\beta_{w(q)}$ will be taken in the form [8]

$$\beta_{w(g)} = \lambda_g \, Nu / 2R , \quad Nu = 0.023 Re^{0.8} Pr^{0.4} .$$

The procedure developed makes it possible to calculate heat transfer from the liquid-gas mixture flowing in the borehole to the frozen rock with allowance for phase changes occurring in the rock and in the presence of solid deposits on the inner walls of the raising column of the borehole.

NOTATION

T, temperature of the gas-liquid flow; $T_0^{(1)}$, $T^{(1,2)}$, T_0, and T_σ, geothermal temperature, melting temperature of the solid rock, temperature of the inner wall of the raising column, and temperature on the surface of the solid phase; t, time; r, radius; β, β_w, coefficients of heat transfer through the system of pipes, and between the gas-liquid flow and the solid phase or the wall of the raising column; R_b and R_0, outer radius of the borehole and inner radius of the raising column; δ_s, thickness of the solid deposits; $R = R_0 - \delta_s$; $\rho^{(i)}$, $T^{(i)}$, $\lambda^{(i)}$, $\chi^{(i)}$. density, temperature, coefficients of thermal conductivity and thermal diffusivity; $i = 1$, parameters of the frozen rock; $i = 2$, parameters of the thawed rock; $l^{(1)}$, heat of fusion of the frozen rock; Q, heat flux per unit length of the borehole; R_* and $R^{(1,2)}$, radii of the thermal effect of the borehole and the thawed zone; δ_{liq}, reduced thickness of the liquid film; $\beta_{w(liq)}$ and $\beta_{w(g)}$, coefficients of heat transfer of the liquid and gas phases; λ_s, λ_{liq}, and λ_g, coefficients of thermal conductivity of the solid, liquid, and gas phases; α, volumetric gas content; Nu, Gr, Pr, Re, Nusselt, Grashof, Prandtl, and Reynolds numbers.

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